Optimal control problem for stochastic higher order Sobolev type equation

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Recently, research on Sobolev type equations has expanded considerably. The incomplete Sobolev type equation

$$Av^{(n)} = Bv + f \tag{1}$$

with the assumption kerA $\neq \{0\}$ has been studied in different aspects for $n \geq 1$ [1]. Here the operators A, B are linear and continuous, acting from Banach space $\mathfrak{V}$ to $\mathfrak{G}$, absolute term $f = f(t)$ models the external force.

The lack of equation (1) with the deterministic absolute term is that, in natural experiments, the system is a subject to random perturbation, for example in the form of white noise. Currently, stochastic ordinary differential equations with various additive random processes are being actively studied [2].

The first results concerning stochastic Sobolev type equations of the first order can be found in [3]. They are based on the extension of the Ito – Stratonovich – Skorokhod method to partial differential equations [4]. In [5] there was studied a stochastic Sobolev type equation of higher order

$$A \overset{o}{\eta}{}^{(n)} = B\eta + w, \tag{2}$$

where w is the stochastic process. It is required to find the random process $\eta(t)$, satisfying (in some sense) equation (2) and the initial conditions

$$\overset{o}{\eta}{}^{(m)}(0) = \xi_m,\ m = 0, 1, \ldots, n-1, \tag{3}$$

where ξ_m are given random variables.

Of particular interest is the optimal control problem. Consider the stochastic Sobolev type equation

$$A \overset{o}{\eta}{}^{(n)} = B\eta + w + Cu, \tag{4}$$

where $\eta = \eta(t)$ is a stochastic process, $\overset{o}{\eta}$ is the Nelson – Gliklikh derivative of process η, $w = w(t)$ is a stochastic process that responds for external influence;

u is unknown control function from the Hilbert space $\mathfrak{U}$ of controls, operator $C \in \mathcal{L}(\mathfrak{U}; \mathfrak{G})$.

Supply (4) with initial Showalter – Sidorov condition

$$P\left(\overset{o}{\eta}^{(m)}(0) - \xi_m\right) = 0, \; m = 0, ..., n-1. \tag{5}$$

We investigate the optimal control problem: search pair $(\hat{\eta}, \hat{u})$, where $\hat{\eta}$ is a solution to problem (4), (5), and the control $\hat{u}$ belongs to $\mathfrak{U}_{ad} \subset \mathfrak{U}$, and satisfies the relation

$$J(\hat{\eta}, \hat{u}) = \min_{(\eta,u)} J(\eta, u).$$

Here $J(\eta, u)$ is some specially constructed penalty functional and $\mathfrak{U}_{ad}$ is a closed convex set in the Hilbert space $\mathfrak{U}$ of controls.

The theorems on the existence and uniqueness of a strong solution to problem (2), (5) and a unique strong solution to problem (4), (5) were proved.

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