NUMERICAL ALGORITHM FOR SOLVING THE PROBLEM OF ESTIMATING THE FLUID PRESSURE UNDER DIFFERENT RANDOM INFLUENCES IN THE BARENBLATT – ZHELTOV – KOCHINA STOHASTIC MODEL

S.A. Zagrebina[1], **A.V. Keller**[1,2], **E.A. Soldatova**[1]
1)Chelyabinsk, South Ural State University,
2) Voronezh, Voronezh State Technical University
zagrebinasa@susu.ru, alevtinak@inbox.ru

Let $G \subset \mathbb{R}^d$ be a bounded domain with a boundary ∂G of class C^∞. Consider in the cylinder $G \times \mathbb{R}_+$ the Barenblatt – Zheltova – Kochina equation [1]

$$(\lambda - \Delta)u_t = \alpha\Delta u + f, \tag{1}$$

with the Dirichlet condition

$$u(x,t) = 0,\ (x,t) \in \partial G \times \mathbb{R}_+. \tag{2}$$

Model (1), (2) simulates the pressure dynamics of the fluid filtered in fractured porous medium with random external influences f [2], processes of moisture transfer in a ground [3] and processes of the solid-to-fluid thermal conductivity in the environment with two temperatures [4]. In equation (1) α and λ are real parameters, that characterize the environment; the parameter $\alpha \in \mathbb{R}_+$, and the parameter λ can also take the negative values [5]; the additive component f presents the random external load [2]. Our goal is to find the random process $u = u(x,t)$ that satisfies (1), (2) with initial Cauchy condition

$$u(x,0) = u_0(x). \tag{3}$$

The problem (1), (2) in corresponding Hilbert spaces can be reduced to the linear stochastic Sobolev type equation

$$Ldu = Mudt + NdW. \tag{4}$$

As an example we consider unstable filtration of the homogeneous fluid in the horizontal layer. The layer is opened by a vertical well of a small radius. At the initial moment pressure of fluid in the layer is constant and is equal to p_0. Assume that a random load begins to influence the fluid. Let the domain $G \subset \mathbf{R}^2$ be presented by disk with the radius r_0 and center on the axis of the well. Introduce a polar system of coordinates and assume that the fluid flow is axially symmetric. In this case we can find the fluid pressure in a fissure if we solve the initial-boundary value problem [1]

$$(\lambda - \Delta_r)p_t = \alpha \Delta_r p + A \sin(\omega t), \tag{5}$$

$$\|p(0,t)\| < +\infty, \quad p(r_0, t) = 0, \quad p(r, 0) = p_0, \tag{6}$$

where $\Delta_r = \dfrac{\partial^2}{\partial r^2} + \dfrac{1}{r}\dfrac{\partial}{\partial r}$, $A \in \mathbf{R}$, $\lambda = 1/\eta$, η is a specific characteristics of the fractured ground, $\alpha = \chi/\eta$, χ is the piezoconductivity coefficient of the fractured ground.

The report will include the algorithm of construction of numerical solution to (5), (6) and simulation results for different random influences, i.e. for different values ω and a random variable A.

References

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