STOCHASTIC BARENBLATT – ZHELTOV – KOCHINA MODEL ON THE SEGMENT WITH WENTZELL BOUNDARY CONDITIONS

N. S. Goncharov[1], **G. A. Sviridyuk**[2]

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Introduction. In terms of the theory of relative *p-bounded operators, we study the stochastic Barenblatt–Zheltov–Kochina model, which describes dynamics of pressure of a filtered fluid in a fractured-porous medium with general Wentzell boundary conditions. In particular, we examine the relative spectrum in the one-dimensional Barenblatt–Zheltov–Kochina equation, and construct the resolving group in the stochastic Cauchy-Wentzell problem with general Wentzell boundary conditions. In the paper, these problems are solved under the assumption that the initial space is a contraction of the space* $L^2(0,1)$.

On the interval $[0,1]$*, let us consider the differential operator*

$$Au(x) = u''(x), \quad x \in [0,1] \tag{1}$$

with the general Wentzell boundary conditions

$$Au(0) + \alpha_0 u'(0) + \alpha_1 u(0) = 0, \tag{2}$$

$$Au(1) + \beta_0 u'(1) + \beta_1 u(1) = 0. \tag{3}$$

By formulas (1)–(3)*, we define the linear operator* $A : dom\ A \subset \mathfrak{F} \to \mathfrak{F}$*. Here* $\mathfrak{F}$ *is the space* $(L^2[0,1], dx\Big|_{(0,1)} \oplus \eta ds\big|_{\{0,1\}})$ *with the norm*

$$\|u\|^2_{\mathfrak{F}} = \int\limits_0^1 |u(x)|^2 dx + \eta_0 |u(0)|^2 + \eta_1 |u(1)|^2,$$

(the full construction of the space $\mathfrak{F}$ *see, for example, in [1]), where* dx *is the Lebesque measure on the interval* $(0,1)$*;* ds *is the point measure at the boundary;* $\eta_0 = \frac{1}{-\alpha_1}, \eta_1 = \frac{1}{\beta_1}$*, where* $\alpha_1 < 0 < \beta_1$ *are positive weights. We consider also the linear manifold* $dom\ A = \{u \in C^2[0,1] :$ *conditions* $(2),(3)$ *are fulfilled*$\}$ *as the domain of the operator* A*. Fix* $\alpha, \lambda \in \mathbb{R}$ *and construct the operators* $L = \lambda - A$ *and* $M = \alpha A$*, where the operator* A *is taken from the considerations above. It is known (see, for example, [2]), that the operators* $L, M \in \mathcal{L}(domA; \mathfrak{F})$ *and the space* $domA$ *is densely embedded in the space* $\mathfrak{F}$*.*

On the interval $[0,1]$*, let us consider the stochastic Barenblatt-Zheltova-Kochina equation*

$$L \overset{\circ}{\eta} (\omega, t) = M\eta(\omega, t) + Nf, \quad (\omega, t) \in [0,1] \times (0,\tau), \tag{4}$$

which describes dynamics of pressure of a filtered fluid in a fractured-porous medium, with the initial Cauchy condition

$$\eta(0) = \xi_0, \tag{5}$$

and the Wentzell boundary conditions

$$\begin{aligned} &\eta_{xx}(0,t) + \alpha_0 \eta_x(0,t) + \alpha_1 \eta(0,t) = 0, \\ &\eta_{xx}(1,t) + \beta_0 \eta_x(1,t) + \beta_1 \eta(1,t) = 0. \end{aligned} \tag{6}$$

[1]South Ural State University, Chelyabinsk, Russia. Email: goncharov.ns.krm@yandex.ru
[2]South Ural State University, Chelyabinsk, Russia. Email: sviridyukga@susu.ru

Here $\eta = \eta(t)$ is a stochastic process on the interval $(0, \tau)$; $\overset{\circ}{\eta}$ is the Nelson–Gliklikh derivative of the process $\eta(t)$; f is a "white noise", which we understand the Nelson–Gliklich derivative an one-dimensional Wiener process (see, for example., [3, 4, 5]); α and λ are the material parameters characterizing the environment; the parameter $\alpha \in \mathbb{R}_+$; the operator $N \in \mathcal{L}(\mathfrak{U}, \mathfrak{F})$ is subject to further clarification.

The purpose of this work is to research the solvability of the problem (4) – (6) *with Wentzell boundary conditions.*

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