

**N. Ratanov** (Chelyabinsk State University). **On the first-passage area functional for telegraph processes**<sup>1</sup>.

For a one-dimensional piecewise linear random process  $X^x(t) = x + \int_0^t c_{\varepsilon(s)} ds$ ,  $\varepsilon(t) \in \{0, 1\}$ , we study the distribution of the first zero crossing time,

$$\mathcal{T}(x) = \inf\{t > 0 \mid X^x(t) = 0\},$$

when the process starts from  $x > 0$ . The distribution of the area between a time axis and the path of the process up to the first return to the origin,

$$\mathcal{A}(x) = \int_0^{\mathcal{T}(x)} X^x(s) ds,$$

is also developed.

The distribution of  $\mathcal{T}(x)$  and  $\mathcal{A}(x)$  are known for the Brownian motion, but have not yet been developed for non-Gaussian processes.

**Theorem 1.** *Moment generating function  $\mathbf{L}(x, q) = E[\exp(-q(\alpha\mathcal{T}(x) + \beta\mathcal{A}(x)))]$  satisfies the differential equation*

$$C\mathbf{L}'(x, q) = [-\Lambda + qu(x)]\mathbf{L}(x, q), \quad x > 0, \quad (1)$$

with the boundary condition

$$\mathbf{L}|_{x=+0} = \boldsymbol{\ell}_q = (\ell_q^{(0)}, \ell_q^{(1)})'. \quad (2)$$

Here  $u(x) = \alpha + \beta x$  and

$$\Lambda = \begin{pmatrix} -\lambda_0 & \lambda_0 \\ \lambda_1 & -\lambda_1 \end{pmatrix}, \quad C = \begin{pmatrix} c_0 & 0 \\ 0 & c_1 \end{pmatrix}.$$

The report will be devoted to the analysis of this equation and its probabilistic consequences.

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<sup>1</sup>The research was supported by the Russian Science Foundation (RSF), project number 24-21-00245, <https://rscf.ru/project/24-21-00245>